Shpaner, Leon

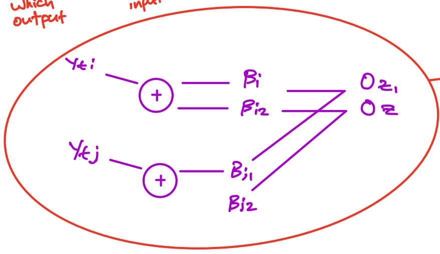
Vector Autoregressive Models (VAR)

regression $\Rightarrow$ $Y_t = \beta_1 z_{t1} + \beta_2 z_{t2} + \beta_3 z_{t3} + w_t + \alpha$

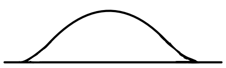
AR $\Rightarrow$ $Y_t = Y_{t-1}$

Multiple outputs $\longrightarrow$ $Y_{ti} = \beta_{i1} z_{t1} + \beta_{i2} z_{t2}$

when　which output　input



$\rightarrow$ taking these z inputs, multiplying them, scaling them w/ this B, adding them, and getting an output Y_{ti}.

$w_t \sim N(\mu, \sigma^2)$  $\longrightarrow$ normal white noise process w/ some variance and mean (univariate) one variable

But, instead of having one variable telling you how wide the distribution is, you're going to have a covariance matrix. You also see the covariance matrix when doing linear regression and/or PCA.

$$E\ w_t w_t' = \Sigma_w$$

$$\begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \sigma_3 & \\ 0 & & & \sigma_4 \end{bmatrix}$$

$\rightarrow$ sigmas are giving you variance of w_t. If you have zeroes on the diagonal, all of your individual components of white noise; this is.
if you calculate this covariance, and have only diagonal, that means that the outputs don't depend on each other.

Autoregressive Model (univariate)

$$x_t = \alpha + \phi_1 x_{t-1} + \phi_{x_{t-2}} + w_t$$

Autoregressive Model (Multivariate)

$$x_t = \alpha + \Phi x_{t-1} + w_t$$

vector　vector　matrix $\rightarrow$ transition　vector　vector

The matrix is telling you how the variables interact. In a regular AR model, your output depends on the past. In a VAR model, your output depends on the past of everything.

You get a vector in, multiply by a matrix, you get a vector out.
Ex. you have a business that goes under and drags the whole economy down w/ it. That's what you can model w/ this.

$$+ \sum_{j=1}^{p} \phi_j x_{t-j}$$

$\longrightarrow$ now you have transition matrix of that lag and our whole vector of time series pushed back by that lag.

G ARCH

auto regressive　conditionally heteroskedastic

You can model when the variance changes.
It is not constant.

Constant variance

AR(1) $\mu_t = E(x_t \mid x_{t-1}, x_{t-2} \dots) = \phi_0 + \phi_1 x_{t-1}$

$\sigma_t^2 = var(x_t \mid x_{t-1}\ x_{t-2} \dots) = var(w_t) = \sigma^2 w.$ $\longleftarrow$ the white noise is the same. It doesn't change at any time step.

ARCH

x_t : value at t

$$r_t = \frac{x_t - x_{t-1}}{x_{t-1}} \approx \log(x_t)$$

$\longrightarrow$ Stock rate return @ time t

Constant $\rightarrow$ $E(x_t \mid \dots)$ $\longrightarrow$ expectation of x_t conditioned on the past, still going to be const
$var(x_t \mid \dots)$ $\longrightarrow$ variance of the return is going to be changing.

$$r_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2$$

Scalar • multiplied by some previous value of the time

constant

We can interpret this as an autoregressive model, except we changed distribution of the variables. So before this was Gaussian and linear, and now it won't be. Let's take the last equation, rewrite it by squaring everything.

$$r_t^2 = \sigma_t^2 \varepsilon_t^2$$ noise squared

$$\alpha_0 + \alpha_1 r_{t-1}^2 = \sigma_t^2$$

$\varepsilon_t \sim N(0,1)$

$$r_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + v_t$$ no dependence on past values of the noise

output @ time t depends on

some constant

some noise term which is no longer Gaussian term, but...

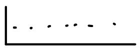
because you have Gaussian terms squared adding them up: $v_t = \sigma_t^2 (\varepsilon_{t-1}^2)$, it actually has chi-squared distribution χ^2

MOST IMPORTANT PART: output depends on constant noise & shifted value of same variable in the past.

ACF $\gamma(h) = \text{cov}(x_{t+h}, x_t)$ → for an autoregressive process, this will always be non-zero.

PACF $\phi_{hh} = \text{corr}(x_{t+h}, x_t) \mid x_{t+1} \cdots x_{t+h-1}$ → autocorrelation on 2 time points separated, but conditional on everything else.

this dependence on the past will not go to zero, but PACF will (partial dependence).

ACF  PACF  } PACF will decay, ACF will not.

$P_r^2 = \alpha_1^h$ for ≥ 0 → looking at PACF of squared returns, it is always non-zero

$var(r_t) = \dfrac{\alpha_0}{1-\alpha_1}$ → w/out any conditioning of the past, the variance is this, which looks constant, but...

$$r_t \mid r_{t-1} \sim N(0, \alpha_0 + \alpha r_{t-1}^2)$$

mean

variance (not constant)

variance of these returns, conditioned on knowing the return previously

$$r_t = \sigma \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2}^2$$ → can have a higher order ARCH model just by adding in more terms.

$(r_t - \mu_t)^2$ → look at the returns minus the mean, and square it, and if you have a model where ACF is high and autocovariance drops off, this is what AR(1) or low order looks like. If the partial is decaying fast, it's an autoregressive model. If it's not (if both ACF & PACF are dropping down, you have something that looks like ARMA model → thus, fit GARCH

$$r_t = \mu_t + \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$ → add-in past version of the noise] GARCH

$$r_t^2 = \alpha_0 + (\alpha_1 + \beta_1) r_{t-1}^2 + v_t - \beta_1 v_{t-1}$$] ARMA(1,1)

BOTTOM LINE: if ACF behaves normally (constant) and PACF behaves normally (tapering off), then fit ARCH model. (AR) If ACF & PACF both tapering off, and not even necessarily @ same rate, then fit GARCH (ARMA) model.